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Is it possible to explain the validity of an inference?

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Since Aristotle we acknowledge with a valid inference that from true premises a true conclusion follows necessarily, or, in other words, that a false conclusion cannot be deduced from true premises. Unfortunately, this notion of validity is compatible with logically false premises or logically true conclusions, and consequently with inferences involving "the paradox of formal implication."[1] It seems that it should be possible to explain how inferences are conceived to be valid. This question has obviously been left unanswered in Kneale's statement: "Logic is concerned with the principles of valid inferences."[2] The introductory remarks by Hintikka and Sandu are similarly vague:

> "It is far from clear what is meant by logic or what should be meant by it. It is nevertheless reasonable to identify logic as the study of inferences and inferential relations. The obvious practical use of logic is in any case to help us to reason well, to draw good inferences. And the typical form the theory of any part of logic seems to be a set of rules of inference. This answer already introduces some structure into a discussion of the nature of logic, for in an inference we can distinguish the input called a premise or premises from the output known as the conclusion. The transition from a premise or a number of premises to the conclusion is governed by a rule of inference. If the inference is in accordance with the appropriate rule, it is called valid."[3]

The "transition from a premise or a number of premises to the conclusion" appears to be not sufficiently clarified if it is merely governed by appropriate rules of inference. What are the basic concepts of the truth of the premises and of its relation to the truth of the conclusion? I think that an instructive answer to this question can be attained (I) by revising the concept of

1 W. Ackermann, „Begründung einer strengen Implikation", *Journal of Symbolic Logic* 21 (1956), (113-128); A. Anderson / N. Belnap, *Entailment: The Logic of Relevance and Necessity,* I-II, I Princeton NJ / London: Princeton Univ. Press, 1975, II (mit J. M. Dunn) Princeton NJ / Oxford: Princeton Univ. Press, 1992; S. Read, *Relevant Logic. A Philosophical Examination of Inference*, New York [i.a.]: Blackwell, 1989.

2 M. and W. Kneale, *The Development of Logic*, Oxford: Clarendon, 1962, 1.

3 J. Hintikka / G. Sandu, ʻWhat is Logic?ʻ, in: D. Jacquette, (ed.), *Philosophy of Logic*, Amsterdam: Elsevier, 2007, (13-39), 13.

logical connectives,[4] (II) by expounding logical connectives as truth value relations, and (III) by showing that inferences are valid because of reflexive truth value relations.

I

In two-valued classical logic it is almost generally agreed that logical connectives appear to be connectives of propositions, which were introduced, for instance, by Barwise and Etchemendy as follows:

> "The connectives $\wedge$, $\vee$ and $\neg$ are truth-functional connectives. Recall what this means: the truth value of a complex sentence built by means of one of these symbols can be determined simply by looking at the truth values of the sentence's immediate constituents. So to know whether P $\vee$ Q is true, we need only know the truth values of P and Q. This particularly simple behavior is what allows us to capture the meanings of truth-functional connectives using truth tables."[5]

Thus, two propositions are connected in formulae like $p \rightarrow q$, $p \wedge q$ and $p \vee q$, which are determined to be true or false by referring to four truth possibilities and truth conditions in corresponding truth tables. Since, on the other hand, the truth values of the formulae $p \rightarrow p$, $\neg (p \wedge \neg p)$ and $p \vee \neg p$ are established only by two truth possibilities and truth conditions, we have difficulties to claim that these formulae are connections of two propositions similar to the former formulae. However, logicians maintain traditionally that p and $\neg p$ *are different* propositions in the law of non-contradiction $\neg (p \wedge \neg p)$ and in the law of excluded middle $p \vee \neg p$.[6] For, according to them, the former law asserts that both p and $\neg p$ cannot together be false, but true, and the latter asserts that both p and $\neg p$ cannot together be true, but false. They deduce from both laws that p and $\neg p$ are contradictories as, for instance, Stebbing points out:

> "It should be observed that both the principle of excluded middle and the principle of contradiction are required to define 'contradictory propositions'. The principle of

4 Cf. S. O. Welding, „Werden die Junktoren der klassischen Logik richtig interpretiert?" (http://www.digibib.tu-bs.de/?docid=00044002)
5 J. Barwise / J. Etchemendy, *Language, Proof and Logic*, Stanford, California: CSLI Publ., 2002, 93.
6 For further discussions: S. Haack, *Deviant logic: some philosophical issues*. London [i.a.]: Cambridge Univ. Press, 1974.

> contradiction alone does not suffice to show that *p* and *-p* are contradictories; they might be contraries."[7]

Similarly, Tarski claims that both principles are necessary to show that p and $\neg p$ are contradictories; for from the principle of non-contradiction alone follows "one of these sentences must be false", and from the principle of excluded middle alone follows "one of the two sentences must be true."[8]

If we take into account that p is the negation of $\neg p$, evidently p and $\neg p$ are both together neither true nor false. Obviously, the assumption that p and $\neg p$ are connected as two different propositions in the formulae $\neg (p \wedge \neg p)$ and $p \vee \neg p$ as well as in $p \rightarrow p$ reveals to be logically inconsistent. On the other hand, if these formulae deal only with one proposition, we do not realize what then is asserted only of p.

However, the doctrine of logical constants analyzed by means of truth functions might have been convincing from the explanation of truth value dependencies described by Whitehead and Russell:

> "It will be observed that the truth values of *p* (*q*, *p* ▪ *q*, […] depend only upon those of *p* and *q*, namely the truth value of "*p* (*q*" is truth if the truth value of either *p* or *q* is truth, and is falsehood otherwise; that of "*p* ▪ *q*" is truth if that of both *p* and *q* is truth, and is falsehood otherwise […]."[9]

In this passage the expression 'the truth values of . . .' does not refer to truth functions, but rather to the truth conditions of a proposition. If we assert that the truth of (i) 'x is greater than y' depends only upon appropriate numbers substituted for 'x' and 'y', we describe only the conditions of truth for the proposition (i). Similarly, if we assert that the truth of (ii) "Jones is a man or Smith is a woman" depends only upon the truth at least of one of the propositions "Jones is a man" and "Smith is a woman", we merely describe the conditions of truth for the proposition (ii). Likewise, if we assert in accordance with Whitehead and Russell that the truth values of (iii) $p \vee q$ depend only upon those of p and q, we obviously merely describe the conditions of truth and of falsehood for the proposition (iii). Clearly, a truth value dependency is only concerned with the truth conditions of a proposition

7 L. S. Stebbing, *A Modern Introduction to Logic*, 7th ed., London: Methuen, 1950, 191.

8 Cf. A. Tarski, *Introduction to Logic and the Methodology of Deductive Sciences*, transl. by O. Helmer. 3rd ed., New York: Oxford Univ. Press, 1965, 135 f. and R. Blanché, *L'Axiomatique*, Paris: Presses Universitaires de France, 1955, 42.

9 A. N. Whitehead / B. Russell, *Principia Mathematica*, vol. I, 2nd ed., Cambridge: Cambridge Univ. Press, 1927, 7 f.

independently of its content.

II

What then is the content of a proposition about logical constants, for instance, about a disjunction? It is worth noting that the propositions (i) – (iii) have in common that each of them is about a two-termed non-functional relation, e.g. between the numbers x and y, the truth values of “Jones is a man” and “Smith is a woman” and between the truth values of p and q, respectively. Thus, the disjunction in (ii) and (iii) reveals itself to consist in a truth value relation. As to this definition of a disjunction in (iii) we should generalize, accordingly, that logical constants such as the affirmation, the negation of p, the conjunction $p \wedge q$, the disjunction $p \vee q$ and the implication $p \rightarrow q$ have to be defined as one- or two-termed non-functional truth value relations. Thus logical constants are neither connectives of propositions nor are they determined by truth functions.

III

Since $p \rightarrow q$ is a proposition about the relation holding between the truth values of p and q, we should conclude that $p \rightarrow p$ asserts that the respective truth value of p implies itself; the repetition of ‘*p*‘ in this formula determines a reflexive relation holding between the truth values of p. Accordingly, we assert in $\neg (p \wedge \neg p)$ that the respective truth value of p is incompatible with its negation, and in $p \vee \neg p$ that the respective truth value of p excludes its negation. Consequently, the formulae $p \rightarrow p$, $\neg (p \wedge \neg p)$ and $p \vee \neg p$ express logically differently the same reflexive relation holding between the truth values only of p.

For developing valid inferences, we are now in the position to construe reflexive relations between the truth values of different propositions. Evidently, the formula

$(p \rightarrow q) \rightarrow (p \rightarrow q)$

is not based only on the truth values of p, but this formula asserts like $p \rightarrow p$ that the respective truth value of $p \rightarrow q$ is identical with itself. On the other hand, a change of this formula will be sufficient for establishing a reflexive relation between the truth values of p

and q for instance in

$(p \rightarrow q \wedge p) \rightarrow q$

The validity of this inference is determined only by the truth of the premises; for if p is true, q must be true, and this is why the truth of the premises contains already the truth of the conclusion. Thus the condition between the truth of the premises and the truth of the conclusion is determined by a reflexive truth-valuable relation. The same method is applied for developing a reflexive relation on the basis of the truth values of more propositions for example in

$(p \rightarrow q \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$

and its various reformulations. However, difficulties arise if the conjunction of the truth values of a proposition is added to the premises of this inference

$(p \rightarrow q \wedge q \rightarrow r \wedge s) \rightarrow (p \rightarrow r)$

Since the condition between the truth of the premises and the conclusion is not completely determined by the reflexive relationship, this inference should be considered not to be valid. The decisive point is this: If the truth values of the premises and the conclusion are partly or completely independent from each other, they cannot establish valid inferences. Thus the reflexive construction of valid inferences leads us to refuse logically false premises or logically true conclusions, and, moreover, logically true premises and logically true conclusions.[10]

10 Cf. S. O. Welding, *Analytische Logik. Die Begründungsstruktur gültiger Schlüsse*, 2nd ed., Münster: LIT Verlag, 2011.